Advances in Engineering Software 32 (2001) 949–955

ADVANCES IN ENGINEERING SOFTWARE

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Computing the kern of a general cross-section

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Received 22 May 2000; revised 17 April 2001; accepted 19 April 2001

Abstract

The 'kern' of a cross-section is the convex region within which any point load applied will produce stresses of the same sign as that of the load throughout the entire cross-section. Based on theorems of Mofid and Yavari [Int. J. Solids Struct. 31 (1998) 2377], an algorithm is presented that can compute the kern of any cross-section numerically. We have developed a program based on this algorithm. A few examples are solved using this program. The results are compared with those of Wilson and Turcotte [Adv. Engng Software 17 (1993) 113] and Mofid and Yavari and excellent agreement is observed. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Core; Bending cross-section

1. Introduction

When a compressive point load is exerted on a section, various points of that section may be set in compression or tension. If this load acts in a special region of a section, the whole section may be set in compression. This special region is called 'the kern' of a section. This concept is widely used in different fields of civil and mechanical engineering. Design of base plates, concrete footings, concrete dams, non-reinforced shells and composite sections often require that tensile stress be prevented and/or controlled. Therefore, the introduction of this zone, the kern, in a general section, in which an axial arbitrary load can be applied, without inducing tensile stress, is very important. The history of kern and the relevant references can be found in Wilson and Turcote [1] and Mofid and Yavari [2].

In this paper, the theorems of Mofid and Yavari [2] are reviewed. These theorems are implemented in a powerful computer program. The program can easily compute the kern of a general cross section and present a graphical demonstration tool. Several examples are solved and some of the results are compared with those of Wilson and Turcotte [1]. The algorithm and the flowchart of this efficient program are given. The paper is organized as follows. Section 2 reviews the theorems of Mofid and Yavari [2]. The algorithm of the program is given in Section 3. A few example problems are presented in Section 4. Conclusions are given in Section 5.

2. Theorems characterizing the kern of a general cross-section

An arbitrary cross-section is shown in Fig. 1; x and y are principle axes and point C is the centroid of this section. The boundary and the domain of this section are denoted by B and D. If a compressive point load, P, is exerted at point A(u, v), the stress at a point M(x, y) is expressed as

$$\sigma(x,y) = -\frac{P}{A}\left(1 + \frac{vy}{r_x^2} + \frac{ux}{r_x^2}\right)$$

where A is the area of cross-section and r_x and r_y are the radii of gyration, with respect to the principal axes. If M(x, y) is a point of the kern, $\sigma(x, y)$ is negative, B^* and D^* show the boundary and domain of the kern, respectively. Thus,

$$1 + \frac{xu}{r_x^2} + \frac{vy}{r_x^2} > 0 \qquad (x,y) \in D^* \ \ \forall (u,v) \in D$$

It means that if a compressive point load, P, is applied at any point of D^*, the stress at all points of D is always compressive. It is another definition of the kern, which is easier to work with. In this part, some theorems, which are directly propounded from Ref. [2], qualify the characteristics of the kern. For the sake of simplicity, it is assumed that

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PII: S0965-9978(01)00019-9

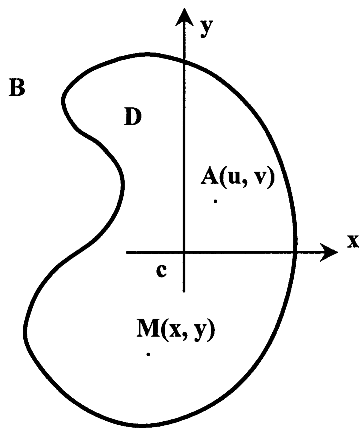

Fig. 1. A general cross-section.

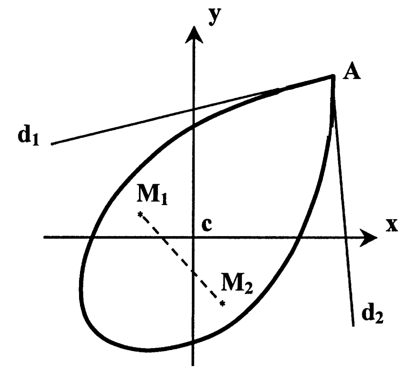

Fig. 3. A cross-section with a cusp.

the section is 'simply connected' and for other groups of sections, the following theorems are generally extended:

Theorem 1. *The centroid of any cross-section belongs to the kern.*

Theorem 2. *If stress at an arbitrary point M, due to a point load, P, exerted to any point of domain B, is always compressive, then the stress at point M, due to a point load, P, applied at any point of D, is also compressive.*

Corollary 1. *If the stress at point M is always zero due to a point load, P, exerted at each point load, P, point M belongs to B^*.*

Definition 1. The smallest convex region of a section with domain D is called 'convex hulls' and is distinguished by CH(D). For any convex section, CH(D) = D, it is clear that in general $D \subseteq$ CH(D).

In Fig. 2, convex hulls of the section is the shaded region. The boundary of the general cross-section can be divided into two sets — the convex part D_{cx}, and the concave part D_{cc}.

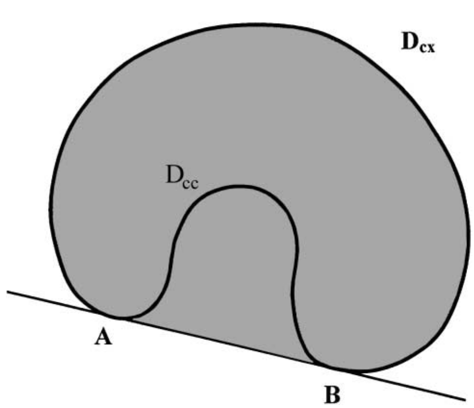

Fig. 2. A non-convex cross-section and its convex hulls.

Theorem 3. *In Theorem 2, D can be replaced by D_{cx}.*

Corollary 2. *For a concave cross-section, we can consider an auxiliary convex cross-section, whose domain is $D' = CH(D)$ with the same area and principal axes as the cross-section. The added area is virtual and A, r_x and r_y remain unchanged. The kern of this auxiliary convex cross-section is the kern of the original cross-section*, i.e. $D'^* = D^*$.

Definition 2. If a compressive point load is applied at point *A*, the stress at point M is denoted by σ_M^A.

Definition 3. If $\sigma_M^A = 0$ and $A \in B$, $M \in B^*$, point M is called the conjugate of point A and the conjugacy is denoted by $M \approx A$.

Definition 4. Each line segment of the boundary of cross-section is called a flat segment of the boundary. The convex part of the boundary B_{cx} can be divided into two parts — the flat part and the non-flat part. The non-flat part is named B_{cx}^n.

Theorem 4. *Each point of B^* is the conjugate of only one point of B_{cx}^n.*

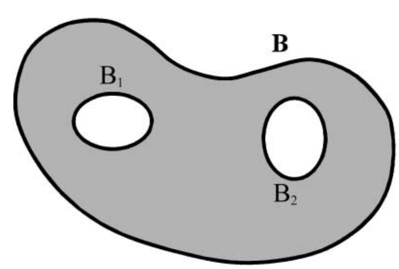

Fig. 4. A multiply connected cross-section.

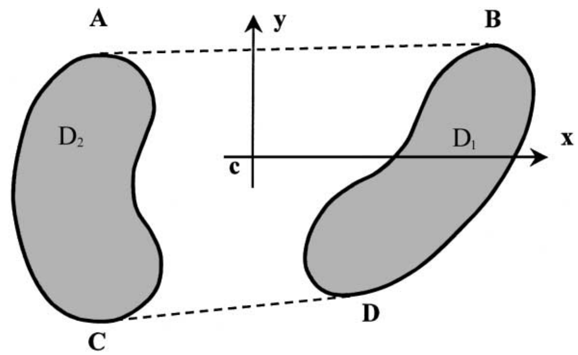

Fig. 5. A disconnected cross-section.

Corollary 3. *All points of every flat segment of the boundary have only one conjugate point on B^*.*

Corollary 4. Suppose that $M \in B^*$ and $M \approx A$. If a point load is applied at point M, the neutral axis of bending due to this load is tangent to B.

Theorem 5. *The kern of a general cross-section is always a convex region.*

Corollary 5. *For a convex cross-section without any flat segment, 'conjugacy' is a one-to-one relation.*

In Fig. 3, a cross-section with a cusp is shown. Most of the sections, which are used, have many cusp, and therefore, this case is important. Suppose that two lines (d_1) and (d_2) are tangent to the boundary of the cross-section at point A, and these lines are the neutral axes of bending due to a point load applied at points M_1 and M_2, respectively.

Theorem 6. *The line segment M_1M_2 belongs to B^*.*

Theorem 7. *The area of the kern of the general cross-section is always nonzero.*

Definition 5. The auxiliary simply connected cross-section of a multiply connected cross-section has the same boundary but all the holes are virtually filled (Fig. 4). The principal axes, the centroid and important values (A, I_x, I_y) of this auxiliary simply connected cross-section are the same as those of the multiply connected cross-section.

Theorem 8. *The kern of the auxiliary simply cross-section D'^* is the same as the kern of a multiply connected cross-section D^*, i.e. $D'^* = D^*$.*

Definition 6. An auxiliary simply connected cross-section of a disconnected cross-section has the same boundary as the convex hulls of the cross-section (Fig. 5). The geometrical properties of the auxiliary cross-section are the same as the disconnected cross-section.

Theorem 9. *The kern of the auxiliary simply connected cross-section of a multiply connected or disconnected cross-section is the same as the kern of the cross-section.*

Definition 7. The kern ratio of a cross-section is distinguished by KR and it is defined as $KR = A^*/A$, where, A and A^* are areas of the cross-section and the kern, respectively. In many conditions, it is possible to encounter moving loads. If the region of the kern is extensive, the cross-section has less possibility of the occurrence of tension.

Corollary 6. *For concave sections, kern ratio (KR) has no maximum value.*

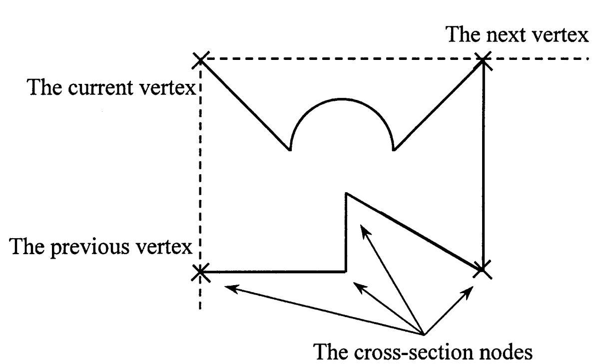

Fig. 6. The previous, current and the next vertex.

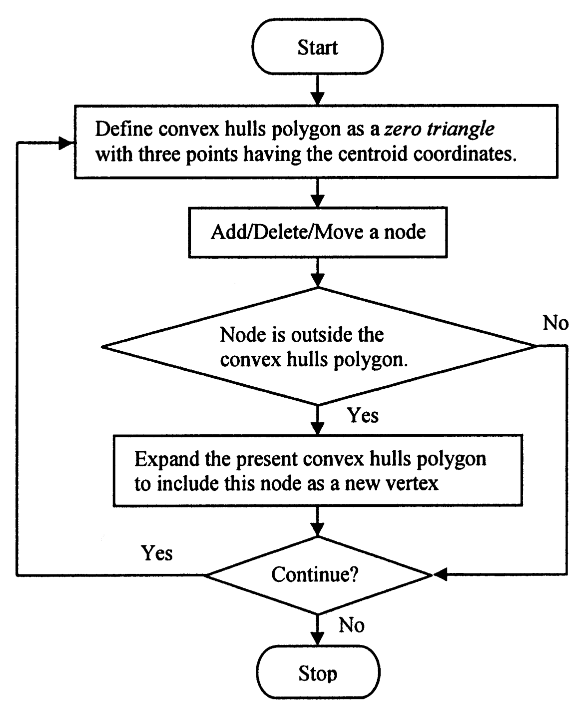

Fig. 7. The working flowchart of the algorithm presented.

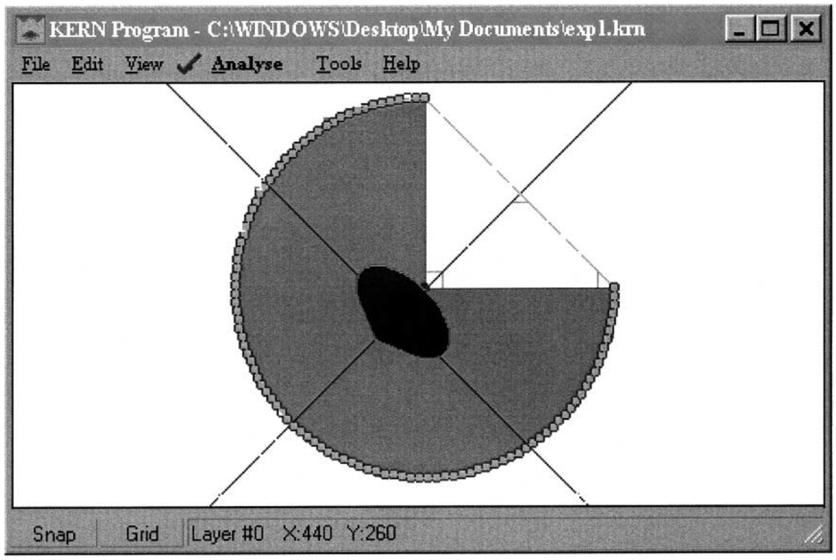

Fig. 8. Example problem (1), Partial section 270° sectorial circular cross-section.

3. Algorithm of the program

Each vertex of the convex hulls has two neighboring vertices which from now on can be called ‘the next’ and ‘the previous’ vertices of the current vertex. This is shown in Fig. 6, where all the convex hulls vertices are concurrent with one or more node(s) of the cross-section polygon. All the other nodes of the cross-section reside in the same side of the line, connecting that node to the current vertex. It is important to note that no direction is defined for distinction between the next and the previous vertices.

When the convex hull is found, the rest of the process of finding the kern is straightforward. Therefore, the kern vertices can be directly found from the intersection of the neutral axes of the successive convex hulls vertices. The step-by-step procedure to find the convex hulls can be outlined as follows:

Find one of the convex hulls vertices. As a matter of fact, this phase may be the most time-consuming part of the algorithm.
Find the next vertex of the convex hulls and call it V.
Compare V with the other known vertices in the list. If it does not exist, add it to the list and repeat the above phase.

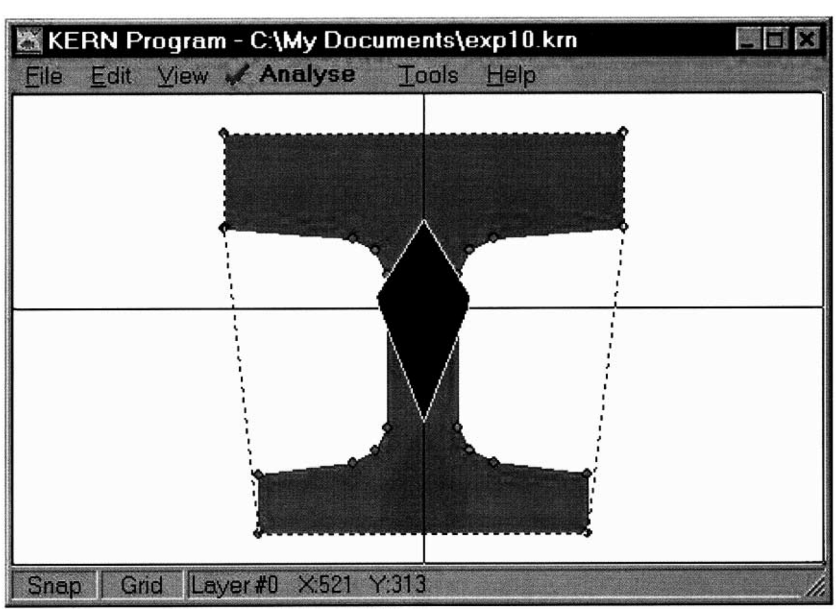

Fig. 9. Example problem (2), I-shaped cross-section.

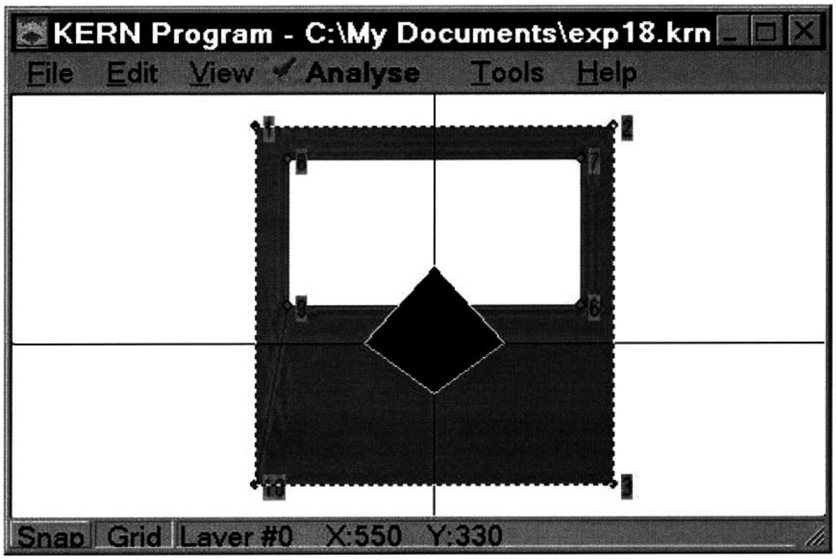

Fig. 10. Example problem (3), hollow-rectangular cross-section.

If V exists in the list, find the previous vertex and compare it with the list. If it exists, the convex hull is found and the procedure can be stopped. Otherwise, the above phases shall be repeated.

Stages of this algorithm are as follows:

1. create/load/edit a section,
2. find the convex hulls of the cross-section,
3. find kerns polygon vertices using the convex hulls and calculate the kern ratio, and
4. display/save/print the results.

It should be noted that finding the convex hulls of a cross-section, although fairly complicated, does not involve huge computations. Fig. 7 shows an appropriate working flowchart of the above-presented algorithm.

4. Example problems

Example 1. To show the capabilities of the program and the method presented, the kern of a partial 270° sectorial circle, which is a relatively complex section and may be of interest in some special structural cases, such as cracked circular column, is shown in Fig. 8. Comparison of the

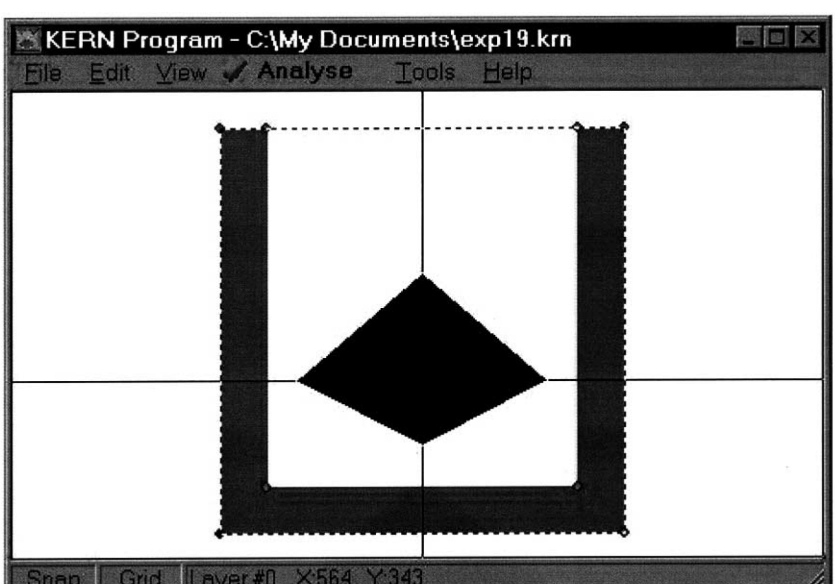

Fig. 11. Example problem (4), U-shaped cross-section.

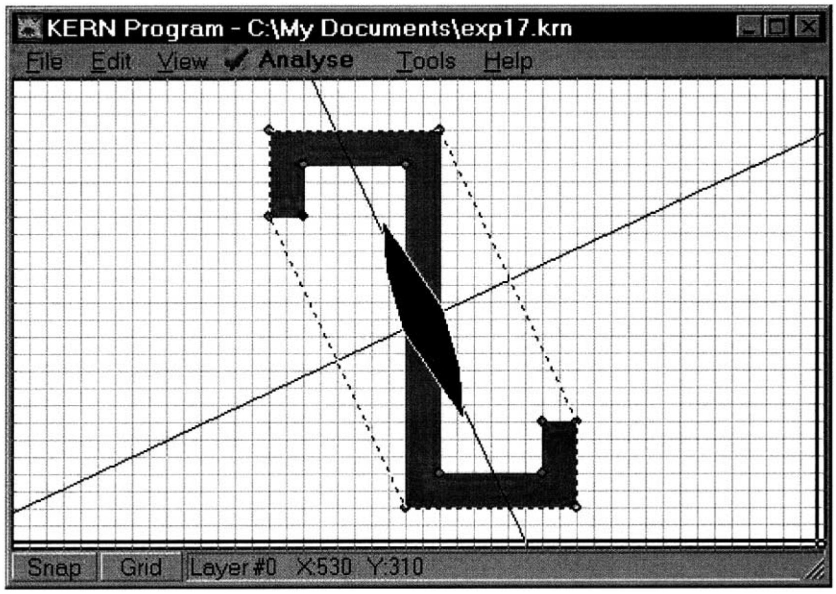

Fig. 12. Example problem (5), Z-shaped cross-section.

results with the solution of the same example problem, presented by Wilson and Turcotte [1], reveals perfect agreement.

Examples 2–7. Kerns of a few cross sections with various geometries are computed in these examples. These cross sections are shown in Figs. 9–14. Some of these cross sections were considered in [1] and [2]. Our results agree with those of [1] and [2].

5. Conclusions

This article explains the algorithm of a computer program that can compute the kern of an arbitrary cross section. The algorithm of this program is based on the theoretical results of Mofid and Yavari [2]. This program has various facilities/tools and is very easy to use. A few examples are solved to show the efficiency of the program. These examples are compared to those of Wilson and Turcotte [1] and Mofid and Yavari [2]. Instead of direct calculation of the kern, our

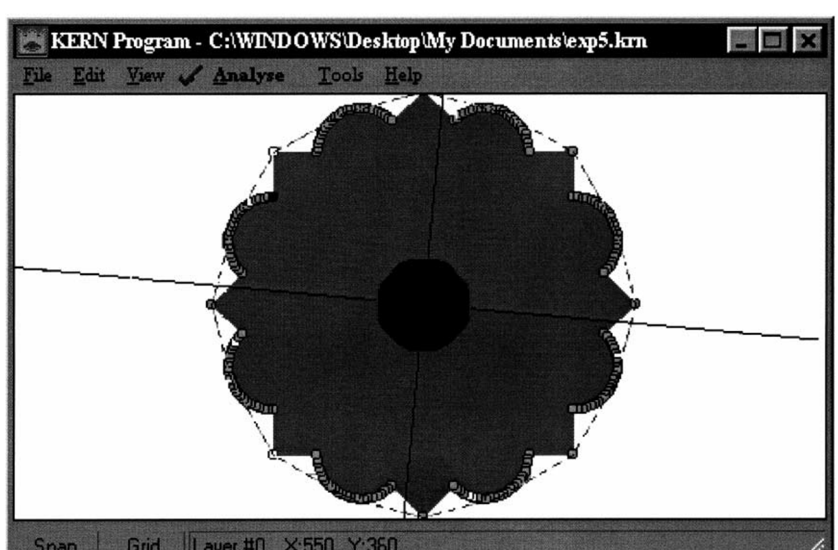

Fig. 13. Example problem (6), flower-shaped cross-section.

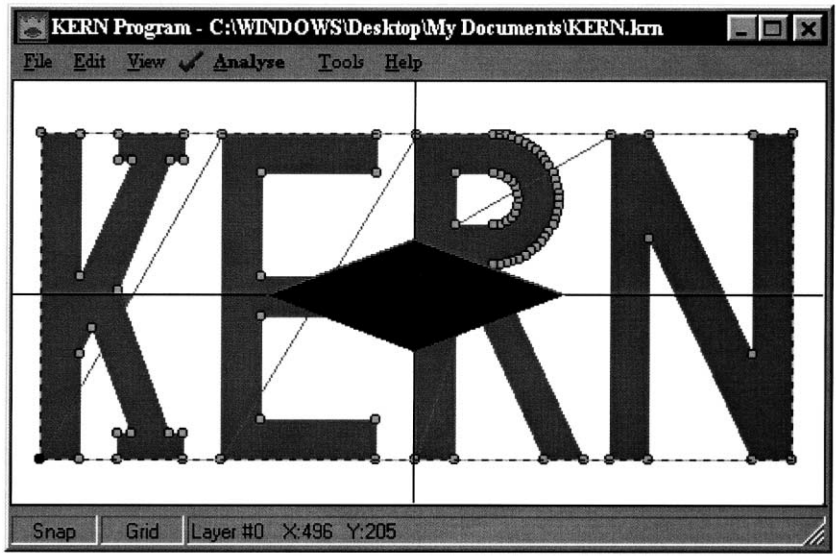

Fig. 14. Example problem (7), a general disconnected shape cross-section.

algorithm uses convex hulls of the cross section. This program is more efficient than that of Wilson and Turcotte. In Wilson and Turcotte's computer program, many coordinates of the boundary of the cross section should be inputted. For example, in the case of the 'flower shape' or the 'KERN shape' hundreds of coordinates should be given to the program as input. However, the present algorithm based on the theorems of Mofid and Yavari [2] does not need to go through this tedious step and is obviously more efficient than that of Wilson and Turcotte. We believe that the program introduced in this paper is very efficient and can help design engineers to find the kern of any cross section.

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[2] Mofid M, Yavari A. On the kern of a general cross-section. Int J Solids Struct 1998;31(17):2377–403.