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Discrete element analysis of dynamic response of Timoshenko beams under moving mass

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Abstract

In this paper, dynamic response of Timoshenko beams under moving mass is analyzed using a numerical method called discrete element technique (DET). In DET, continuous flexible beam elements are replaced by a system of rigid bars and flexible joints. We present a DET model of Timoshenko beams under moving mass. The results of our DET model are compared with the solutions obtained by PAFEC (programs for automatic finite element calculations) for Euler–Bernoulli beams and finite difference method for Timoshenko beams. The effects of beam thickness and moving mass velocity on dynamic response of beams under moving mass are numerically studied.

1. Introduction

One of the most important problems facing structural and design engineers is the analysis of dynamic behavior of bridges subjected to moving loads (moving forces and moving masses). This is one of the oldest problems in structural dynamics. In recent years, the speed and weight of commercial vehicles have been increased significantly. However, due to economical requirements, the bridge structures carrying these vehicles are fabricated much lighter. These structures are therefore subjected to severe vibrations and dynamic stresses, which in turn are much more than the corresponding static stresses.

Historically, the analysis of moving loads on bridges goes back to the 19th century, when railroad construction was first initiated. Research on this subject is still in progress, especially due to the development of numerical techniques for solving complicated differential equations and also due to the improvement of computers. Since the middle of the 19th century, when railway construction began, the problem of oscillation of bridges under traveling loads has interested many engineers [1]. Timoshenko [2] considered the problem of a pulsating load passing over a bridge, while Inglis [3] performed an analysis on trains crossing a bridge and considered many important factors such as the effect of moving load, the influence of damping and suspension of locomotives.

The problem of a concentrated force moving with a constant velocity along a beam, when neglecting damping forces, was solved by Timoshenko [4] and an expression for the critical velocity of a moving mass was found. The dynamic analysis of a simply supported beam carrying a moving mass was performed by Stanisic and Hardin [5]. A comprehensive treatment of the subject for the vibration of structures resulting from moving loads has been given by Frýba [6]. The vibration of an Euler–Bernoulli beam traversed by uniform partially distributed moving mass was studied by Esmailzadeh and Ghorashi [7].

Mofid [8] (see also Ref. [9]) developed a new method for solving the problem of vibration of Euler–Bernoulli beams subjected to moving mass. Here we call the method discrete element technique (DET). It is a simple and fast method for solving this problem, but its application is limited to the case of Euler–Bernoulli beams. The present work extends the scope of previous study [8,9] by considering the Timoshenko theory for the beam and considering the effects of both shear deformation and rotary inertia. DET is used for determining the dynamic response of beams with different boundary conditions subjected to moving mass. The critical velocity for a moving mass to get maximum beam dynamic deflection is numerically calculated. The present numerical results also confirm that the deflection under the moving load is not always an upper bound solution for the moving

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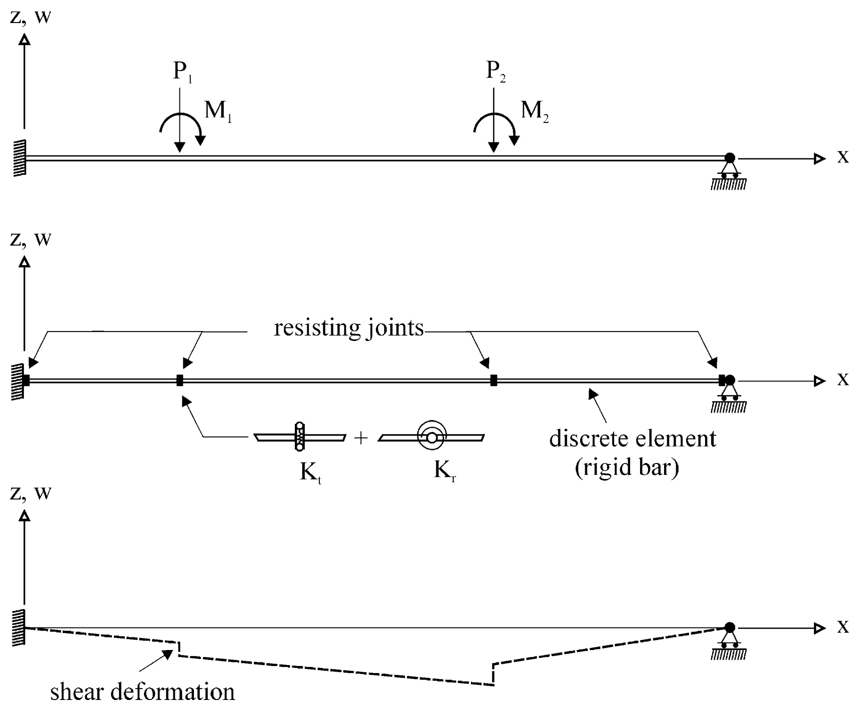

Fig. 1. A discrete element beam approximation.

mass formulation for both the Euler–Bernoulli and Timoshenko beams.

In this paper, the following assumptions are made. First, the beam is assumed to be of constant cross-section with uniform mass distribution. Furthermore, its dynamic characteristics are described by the Timoshenko beam equations. Second, the effects of inertia for both the beam and the moving mass are taken into account with the gravitational effect of load. Third, the load moves with a constant speed and is guided in such a way that it is in contact with the beam at all times. The objectives of this investigation are: (1) to present a simple and practical technique for determining the dynamic response of a Timoshenko beam, with various essential boundary conditions, carrying a moving mass, (2) to formulate the solution of the problem in the general form, (3) to verify the solutions by applying them to some test problems and comparing the results with those from literature, and (4) to study the important factors such as moving mass velocity and beam thickness in the moving mass problem. This paper is organized as follows. In Section

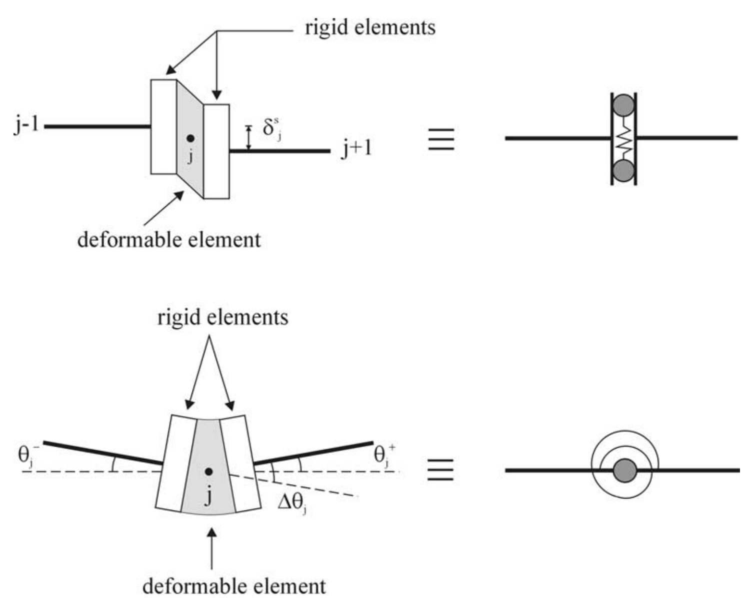

Fig. 2. Typical shearing and bending joints.

2, DET is generalized for Timoshenko beams. A DET model of Timoshenko beams under moving mass is presented in Section 3. Some numerical examples are solved in Section 4. The conclusions are given in Section 5.

2. Discrete element model for Timoshenko beams

In this model the flexible beam is replaced by a system of rigid bars and joints that resist relative rotation of attached bars (Euler–Bernoulli beam model) or both relative rotation and relative deflection of attached bars (Timoshenko beam model). For example, we may replace the fixed-hinged beam by the three-bar system for an approximate analysis as shown in Fig. 1. All deformations take place at the joints. The deflection of joints is the sum of bending and shear deflections. Bending deflection is due to bending moments and shearing deflection is due to shear forces. The flexibility of the joints (both bending and shearing) is chosen such that under uniform bending moment the model gives the exact bending deformations and under uniform shear force, the model gives the exact shearing deformations. In any problem when the number of rigid elements increases, the moment and the shear force changes between adjacent joints decrease and hence we reach uniform moment and uniform shear force over a short length. Thus, the results improve as we use more rigid elements and the results converge to the exact solution as the number of discrete elements increases. However, the numerical solution of these systems will be affected by additional discretizations and round-off errors. In the development of the model, deflection of joints due to bending deformation and shearing deformation must be considered separately. A typical joint that can have only bending deformation is shown in Fig. 2. It is composed of a deformable centerpiece and a rigid element on each side. In the same figure, a typical shear deformation joint is shown. It is composed of a deformable centerpiece and a rigid element on each side.

In the case of bending deformation we have

$$M_j = K_j \theta_j \tag{1}$$

where M_j is the bending moment, θ_j is the relative rotation of attached bars, and K_j is the rotational stiffness of the joint. Here we use the theorem of virtual work to develop an expression for K_j. Consider the beam element under the action of uniform moment shown in Fig. 3(a). Moment

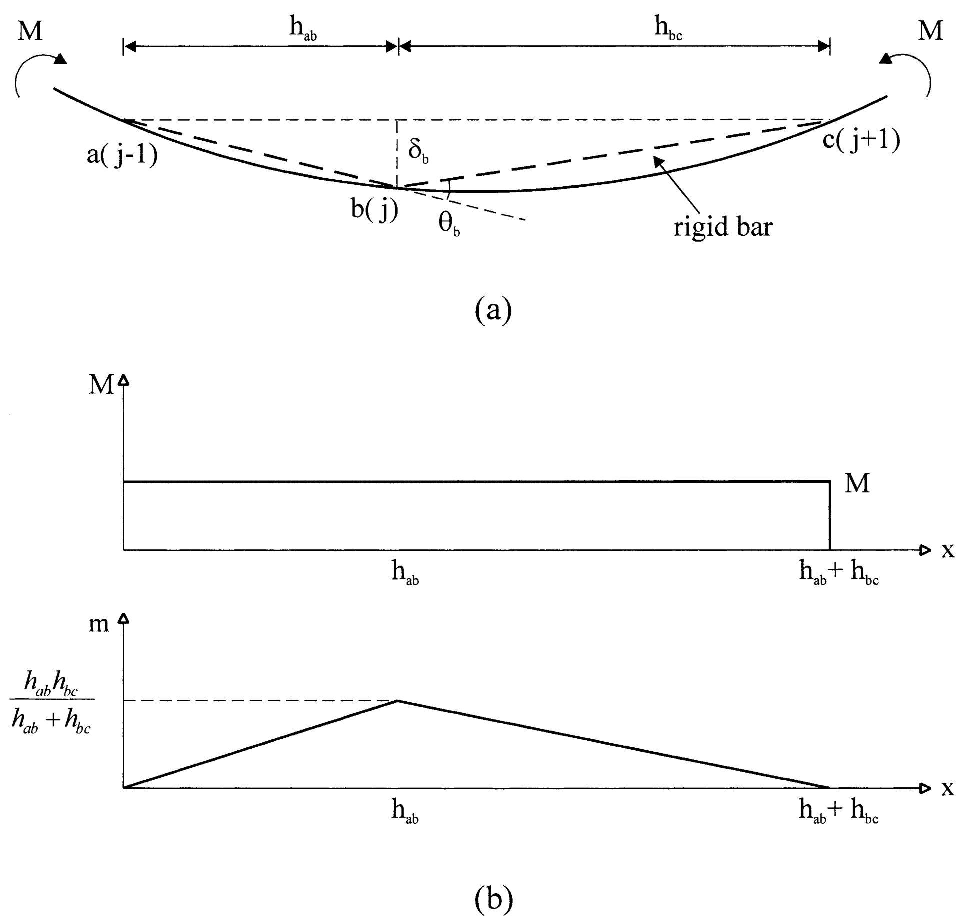

Fig. 3. (a) Discrete element approximation for a bending element, (b) moment diagrams.

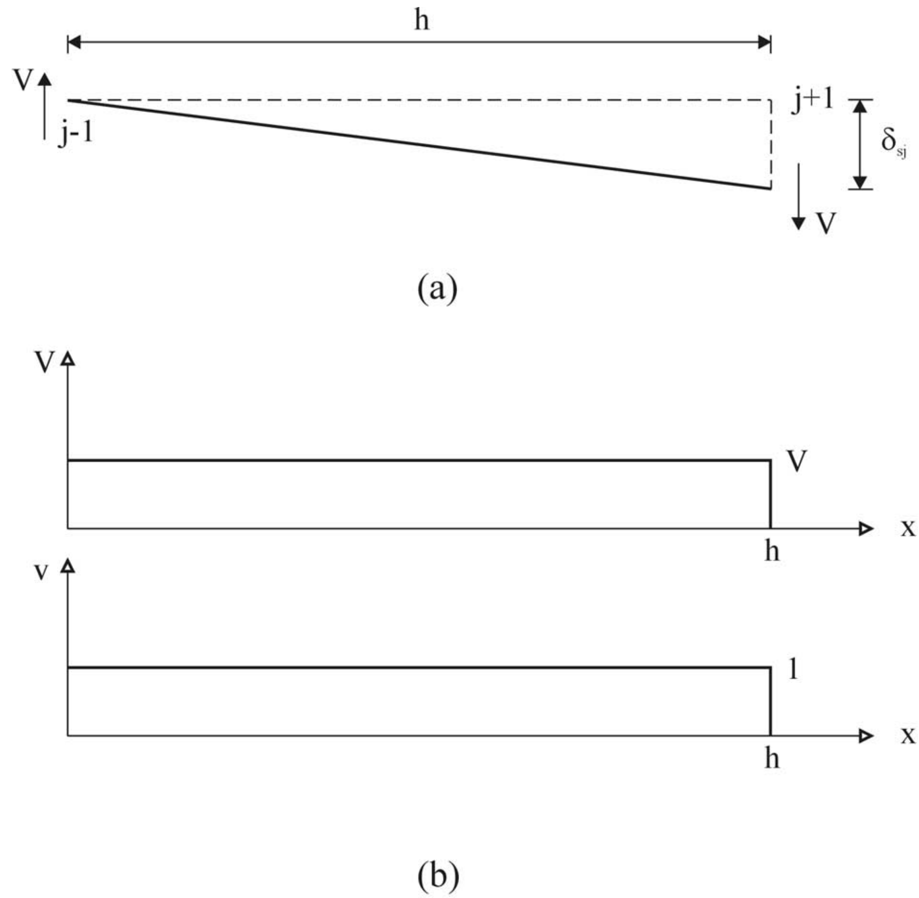

Fig. 4. (a) Discrete element approximation for a shear element, (b) shear diagrams.

diagrams for external uniform moment (M) and unit load in intermediate joint (m) are shown in Fig. 3(b). Using virtual work theorem we have

$$\delta_b = \int_0^{h_{ab}} \frac{Mm}{EI}\,dx + \int_0^{h_{bc}} \frac{Mm}{EI}\,dx \tag{2}$$

If the lengths of the bars are equal, we obtain

$$\delta_b = \frac{Mh^2}{2EI} \tag{3}$$

Therefore,

$$\theta_b = \frac{Mh}{EI} \tag{4}$$

Hence, for interior joints we obtain

$$K = \frac{EI}{h} \tag{5}$$

The expression for a fixed end is

$$K_{fixed} = \frac{2EI}{h} \tag{6}$$

Similarly, for shear deformation we have

$$V_j = K_{sj}\delta_{sj} \tag{7}$$

where V_j is the shear force, δ_{sj}, the relative deflection of attached bars, and K_{sj} is the translational stiffness of the joint. Consider the beam under the action of a uniform shear force shown in Fig. 4(a). Shear diagrams for external uniform shear (V) and unit shear in joints (v) are shown in Fig. 4(b). Using virtual work theorem we have

$$\delta_{sj} = \int_0^{h} \frac{\alpha V v}{GA}\,dx \tag{8}$$

where α is the shear coefficient. Thus

$$\delta_{sj} = \frac{\alpha h}{GA} V \tag{9}$$

Thus, the shearing stiffness of the joint is

$$K_{sj} = \frac{GA}{\alpha h} \tag{10}$$

3. Formulation of the problem of a Timoshenko beam under moving mass

To analyze the response of a Timoshenko beam subjected to a moving mass, we consider the bending and shearing deformations separately. To formulate this problem for different boundary conditions, we have introduced three

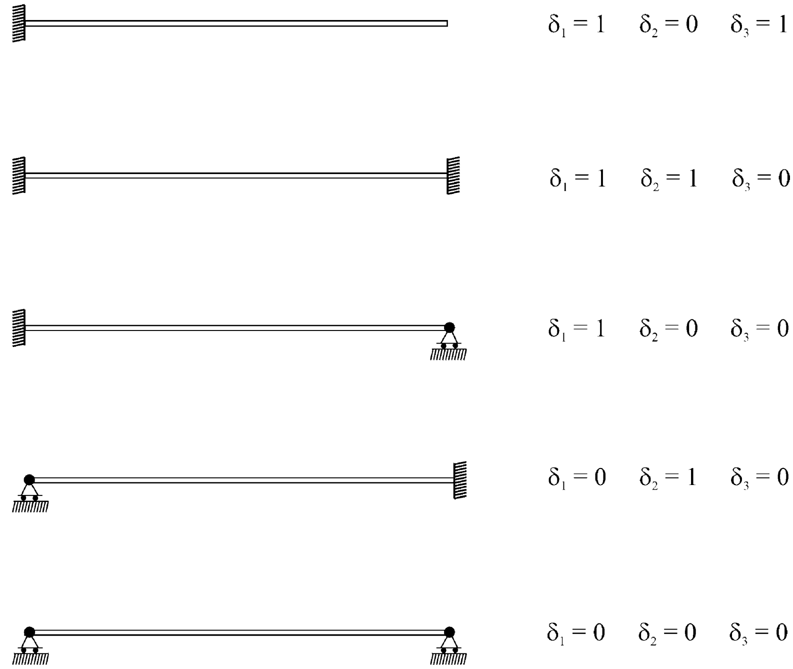

Fig. 5. Boundary condition parameters.

parameters δ_i $(i = 1, 2, 3)$ to define the type of the boundary conditions (fixed, hinged or free). The parameter δ_1 corresponds to the left end of the beam and the parameters δ_2 and δ_3 correspond to the right end of beam. Parameters δ_1 and δ_2 present contribution of the corresponding strain energy resulting from fixed end joints and δ_3 indicates contribution of the corresponding kinetic energy and external work resulting from the movement of the free end. While formulating the problem it is assumed that the first node is restricted to be 'fixed' or 'hinged' only and that it cannot be 'free'. However, if the first node was free, with the change in direction of moving mass, the same formulation is still applicable. For the case of shearing deformation, there is only one boundary parameter, δ, which is similar to δ_3. The values, which are assigned to each of these parameters, are shown in Fig. 5.

Based on the assumption of a constant inertia, the total energy of motion of the beam under the moving mass M, neglecting the damping and rotary inertia of moving mass, can be written in two parts: 'the total energy of beam' and 'the total energy affected by moving mass'. For the sake of simplicity, we consider the bending and shearing deflections of the beam separately. For the bending deflection, the following expressions are valid for energies of the n nodes (see Refs. [8] and [10] for more details):

$$T_{\mathrm{I}} = \frac{1}{2}\sum_{j=1}^{n} m_j^*\left(\frac{\mathrm{d}y_j^*}{\mathrm{d}t}\right)^2 + \frac{1}{2}\sum_{j=1}^{n} I_j^*\omega_j^2 \tag{11}$$

$$U_{\mathrm{I}} = \frac{1}{2}\sum_{j=1}^{n} M_j\theta_j = \frac{1}{2}\sum_{j=1}^{n} K_j\theta_j^2 \tag{12}$$

$$\Omega_{\mathrm{I}} = \sum_{j=1}^{n-1} p_j y_j + \delta_3 p_n y_n \tag{13}$$

$$V_{\mathrm{I}} = U_{\mathrm{I}} - \Omega_{\mathrm{I}} \tag{14}$$

$$\Psi_{\mathrm{I}} = T_{\mathrm{I}} + V_{\mathrm{I}} \tag{15}$$

$$M_j = K_j\theta_j, \qquad K = \frac{EI}{h}, \qquad m_j^* = mh, \qquad h = \frac{L}{n},$$
$$\theta_j = \frac{1}{h}[(y_j - y_{j-1}) + (y_j - y_{j+1})], \qquad I_j^* = \frac{mh^3}{12}\left(1 + \frac{b^2}{h^2}\right),$$
$$\omega_j = \frac{1}{h}(\dot{y}_j - \dot{y}_{j-1}), \qquad y_j^* = \frac{1}{2}(y_{j-1} + y_j) \tag{16}$$

Substituting Eq. (16) in Eqs. (11) and (12), and applying the boundary conditions specified in Fig. 5 yields

$$T_{\mathrm{I}} = \left(\frac{mL}{6n}\right)\left(1 + \frac{b^2}{2h^2}\right)\left[2\sum_{j=1}^{n-1}\dot{y}_j^2 + \delta\dot{y}_n^2\right] + \left(\frac{mL}{6n}\right) \times\left(1 - \frac{b^2}{2h^2}\right)\left[\sum_{j=1}^{n-2}\dot{y}_j\dot{y}_{j+1} + \delta\dot{y}_{n-1}\dot{y}_n\right] \tag{17}$$

$$V_{\mathrm{I}} = \frac{EIn^3}{2L^3}\left[2\delta_1(-y_1)^2 + (2y_1 - y_2)^2 + \sum_{j=1}^{n-2} \times(-y_{j-1} + 2y_j - y_{j+1})^2 + (-y_{n-1} + 2y_{n-1} - \delta_3 y_n)^2 + 2\delta_2(-y_{n-1} + \delta_3 y_n)^2\right] - \sum_{j=1}^{n-1} p_j y_j - \delta_3 p_n y_n \tag{18}$$

The total energy affected by moving mass Ψ_{II}, can be written as

At the first node:

$$T_{\mathrm{II}} = \frac{1}{2}Mc^2, \qquad \Omega_{\mathrm{II}} = 0 \;\; 0 < t_0 < \frac{L}{2nc} \tag{19a}$$

At the interior nodes:

$$T_{\mathrm{II}} = \frac{1}{2}M\left[\frac{\mathrm{d}y_j}{\mathrm{d}t}\right]^2 + \frac{1}{2}Mc^2,$$

$$\Omega_{\mathrm{II}} = Mgy_j \;\; \frac{(2j-1)L}{2nc} < t_j < \frac{(2j+1)L}{2nc}, \tag{19b}$$

$j = 1, 2, \ldots, n-1$

At the final node:

$$T_{\mathrm{II}} = \frac{1}{2}\delta_3 M\left[\frac{\mathrm{d}y_n}{\mathrm{d}t}\right]^2 + \frac{1}{2}Mc^2,$$

$$\Omega_{\mathrm{II}} = \delta_3 Mgy_n \;\; \frac{(2n-1)L}{2nc} < t_n < \frac{L}{n} \tag{19c}$$

In summary

$$U_{\mathrm{II}} = \sum_{j=1}^{n} \text{Strain energy of moving mass} \tag{20}$$

$$V_{\mathrm{II}} = \sum_{j=1}^{n} \text{Potential energy} = -\Omega_{\mathrm{II}} \tag{21}$$

$$\Psi_{\mathrm{II}} = T_{\mathrm{II}} + V_{\mathrm{II}} \tag{22}$$

The dynamic equation for this discrete system may be obtained from Lagrange's equations. For a conservative system we have

$$\left[\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{y}_j}\right) + \left(\frac{\partial V}{\partial y_j}\right)\right] = 0 \tag{23}$$

From Eqs. (15) and (22), Eq. (23) becomes

$$\left[\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{y}_j}\right) + \left(\frac{\partial V}{\partial y_j}\right)\right]_{\mathrm{I}} + \left[\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{y}_j}\right) + \left(\frac{\partial V}{\partial y_j}\right)\right]_{\mathrm{II}} = 0 \tag{24}$$

Substituting Eqs. (17)–(19c) in Eq. (24) and rearranging in matrix form results in

$$(\mathbf{Q} + \mathbf{H})\mathbf{Y}_{,tt} + \mathbf{AY} = \mathbf{W} + \mathbf{P} \tag{25}$$

where $\mathbf{P}$ is generalized loading, $\mathbf{W}$, the weight of the moving mass and $\mathbf{Y}$, the displacement vector. $\mathbf{H}$ has a single time-dependent diagonal term, $\mathbf{Q}$ is a constant symmetric tridiagonal matrix, which is the sum of two other matrices and $\mathbf{A}$ is a constant symmetric pentadiagonal matrix. Rearranging Eq. (25) in the form of a matrix at an interior node (j), we obtain

$$[\mathbf{Q} + \mathbf{H}_j]\{\mathbf{Y}_{,tt}\} + [\mathbf{A}]\{\mathbf{Y}\} = [\mathbf{W}_j + \mathbf{P}]$$
$$\frac{(2j-1)L}{2nc} < t_j M < \frac{(2j+1)L}{2nc} \tag{26a}$$

And at the end node (n) we have

$$[\mathbf{Q} + \mathbf{H}_n]\{\mathbf{Y}_{,tt}\} + [\mathbf{A}]\{\mathbf{Y}\} = [\mathbf{W}_n + \mathbf{P}]$$
$$\frac{(2n-1)L}{2nc} < t_n < \frac{L}{c} \tag{26b}$$

where

$$\{\mathbf{Y}\}^{\mathrm{T}} = \{y_1\, y_2 \cdots y_n\}, \qquad \{\mathbf{Y}_{,tt}\}^{\mathrm{T}} = \{y_{1,tt}\, y_{2,tt} \cdots y_{n,tt}\} \tag{27a}$$

$$\begin{aligned}
\{\mathbf{W}_1 + \mathbf{P}\}^{\mathrm{T}} &= \{(w_1 + p_1)p_2 \cdots p_{n-1}\delta_3 p_n\} \\
\{\mathbf{W}_2 + \mathbf{P}\}^{\mathrm{T}} &= \{w_1(w_2 + p_2) \cdots p_{n-1}\delta_3 p_n\} \\
&\cdots\cdots \\
\{\mathbf{W}_n + \mathbf{P}\}^{\mathrm{T}} &= \{p_1\, p_2 \cdots p_{n-1}\delta_3(w_n + p_n)\}
\end{aligned} \tag{27b}$$

$$[\mathbf{Q}] = \frac{mL}{6n}\begin{bmatrix} 4 & 1 & & & & & \\ 1 & 4 & & & & & \\ & & \cdots & \cdots & & & \\ & & \cdots & \cdots & & & \\ & & & & 4 & 1 & \\ & & & & 1 & 4 & \delta_3 \\ & & & & & \delta_3 & 2\delta_3 \end{bmatrix} + \frac{b^2}{4h^2}\begin{bmatrix} 4 & -2 & & & & & \\ -2 & 4 & & & & & \\ & & \cdots & \cdots & & & \\ & & \cdots & \cdots & & & \\ & & & & 4 & -2 & \\ & & & & -2 & 4 & -2\delta_3 \\ & & & & & -2\delta_3 & 2\delta_3 \end{bmatrix} \tag{27c}$$

$$[\mathbf{A}] = \frac{EIn^3}{2L^3}\begin{bmatrix} 4\delta_1+10 & -8 & 2 & & & & & & & \\ -8 & 12 & -8 & 2 & & & & & & \\ 2 & -8 & 12 & -8 & 2 & & & & & \\ & 2 & -8 & 12 & -8 & 2 & & & & \\ & & & & \cdots & \cdots & & & & \\ & & & & \cdots & \cdots & & & & \\ & & & & 2 & -8 & 12 & -8 & 2 & \\ & & & & & 2 & -8 & 12 & -8 & 2\delta_3 \\ & & & & & & 2 & -8 & 4\delta_2+10 & -4\delta_3(1+\delta_2) \\ & & & & & & & 2\delta_3 & -4\delta_3(1+\delta_2) & 2\delta_3(1+2\delta_2) \end{bmatrix} \quad (27\text{d})$$

$$[\mathbf{H}_1] = \begin{bmatrix} M & 0 & & & \\ 0 & 0 & & & \\ & & \cdots & & \\ & & & 0 & 0 \\ & & & 0 & 0 \end{bmatrix},$$

$$[\mathbf{H}_2] = \begin{bmatrix} 0 & 0 & & & \\ 0 & M & & & \\ & & \cdots & & \\ & & & 0 & 0 \\ & & & 0 & 0 \end{bmatrix}, \ldots, \quad (27\text{e})$$

$$[\mathbf{H}_n] = \begin{bmatrix} 0 & 0 & & & \\ 0 & 0 & & & \\ & & \cdots & & \\ & & & 0 & 0 \\ & & & \delta_3 & M \end{bmatrix}$$

For the case of shearing deflection, the matrix $[\mathbf{A}]$ should be replaced by $[\mathbf{A}_s]$ (see Eq. (31)). Strain energy of the beam in pure shear is

$$U_{sI} = \frac{1}{2}\sum_{j=1}^{n} K_{sj}\delta_{sj}^2 \quad (28)$$

where K_{sj} is the shear stiffness of joints Eq. (10) and δ_{sj} is

$$\delta_{sj} = y_j - y_{j-1} \quad (29)$$

Hence

$$U_{sI} = \frac{GA}{2\alpha h}\left[\sum_{j=2}^{n-1}(y_j - y_{j-1})^2 + (\delta y_n - y_{n-1})^2\right] \quad (30)$$

Similar to the pure bending formulation, in pure shear we have

$$[\mathbf{A}_s] = \frac{GA}{2\alpha h}\begin{bmatrix} 4 & -2 & & & & & \\ -2 & 4 & -2 & & & & \\ & -2 & 4 & & & & \\ & & & \cdots & & & \\ & & & & 4 & -2 & \\ & & & & -2 & 4 & -2\delta \\ & & & & & -2\delta & 2\delta \end{bmatrix} \quad (31)$$

where $\delta = \delta_3$. Matrices $[\mathbf{Q}]$, $[\mathbf{H}]$, $\{\mathbf{Y}_{,tt}\}$, $\{\mathbf{Y}\}$, $[\mathbf{W}]$ and $\{\mathbf{P}\}$ are similar to the previous ones. The nodal deflections due to pure shear can be obtained from Eq. (32).

$$(\mathbf{Q} + \mathbf{H})\mathbf{Y}_{,tt} + \mathbf{A}_s\mathbf{Y} = \mathbf{W} + \mathbf{P} \quad (32)$$

For obtaining deflection of a Timoshenko beam, we should add the nodal deflections from Eqs. (25) and (32).

Problem 1 (reference [1])

Problems 2 and 3 (reference [8])

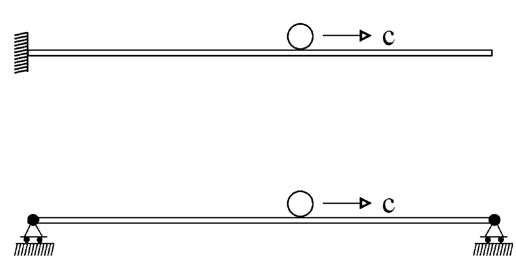

Fig. 6. Sample beams.

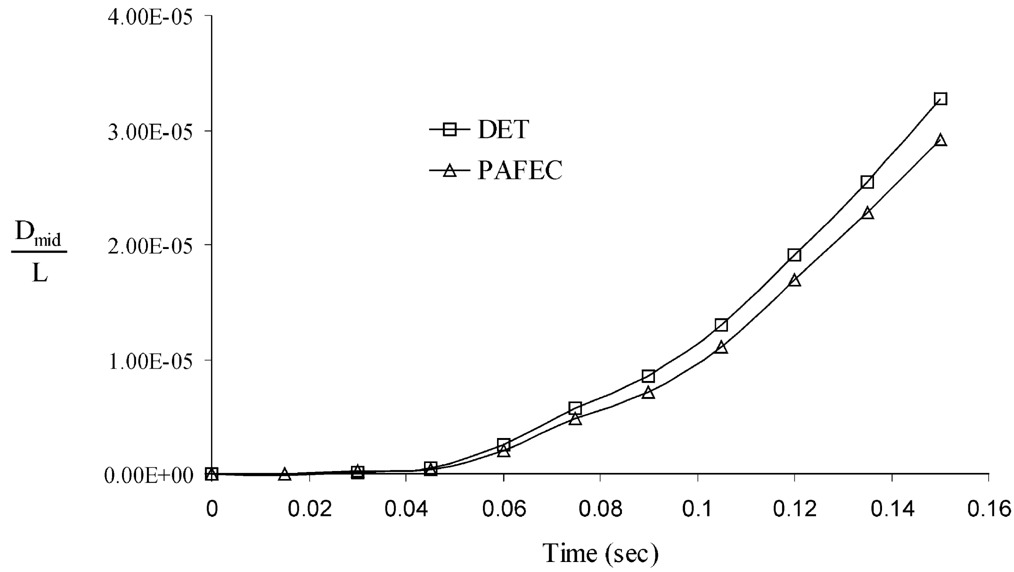

Fig. 7. Dynamic deflection of end point of Beam 1.

4. Numerical examples and discussion

Three examples are solved in this section. The data for each problem are as follows:

Problem 1. Beam 1 is fixed-free (see Fig. 6) with the following characteristics:

$M = 3$ lb s^2/in., $c = 2000$ in./s, $m = 2$ lb s^2/in.,

$E = 3 \times 10^7$ psi, $G = 1.15 \times 10^7$ psi, $I = 110$ in.^{4},

$A = 9.13$ in.^{2}, $L = 300$ in., $\alpha = 1.2$

where M is the moving mass, m, the total mass of the beam, c, the moving mass velocity, E, the Young's modulus, G, the shear modulus, I, the moment of inertia, A, the beam cross-sectional area, L, the beam length, and α, the shear factor.

Problem 2. Beam 2 is simply supported (see Fig. 6) with the following characteristics

$M = 21.83$ kg, $c = 27.49$ m/s, $m = 87.04$ kg,

$E = 2.02 \times 10^{11}$ N/m^2, $G = 7.7 \times 10^{10}$ N/m^2,

$I = 5.71 \times 10^{-7}$ m^4, $A = 1.31 \times 10^{-3}$ m^2,

$L = 4.352$ m, $\alpha = 1.43$.

Problem 3. Beam 3 is similar to Beam 2 except for the

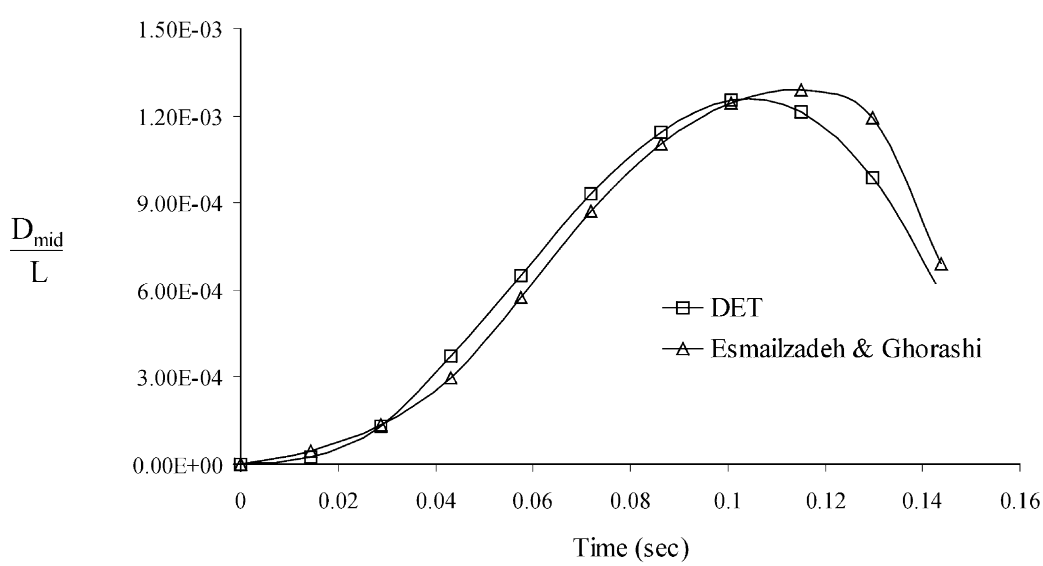

Fig. 8. Dynamic deflection of mid-point of Beam 2.

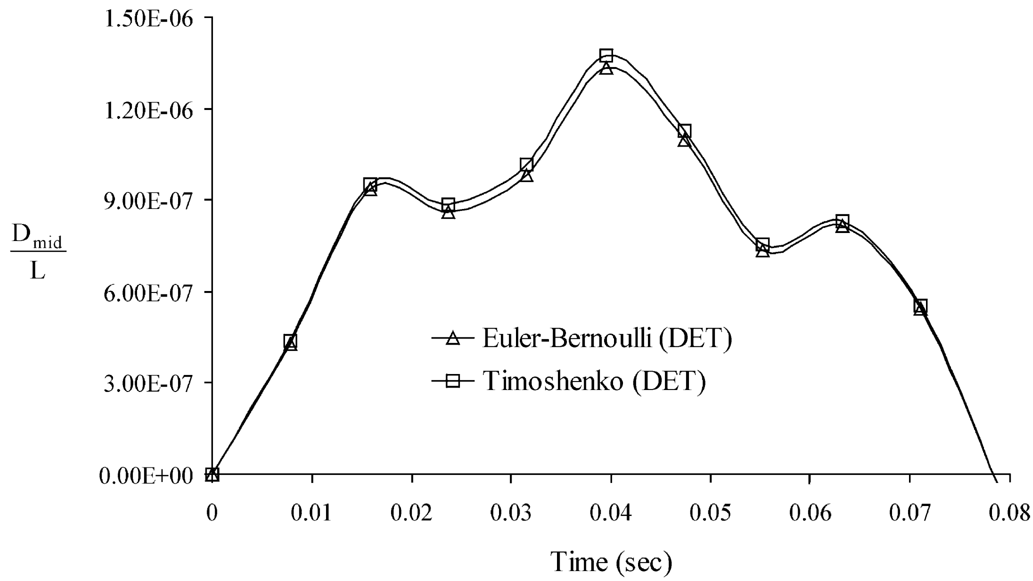

Fig. 9. Dynamic deflection of mid-point of Beam 3.

following different parameters

$M = 200$ kg, $\quad c = 50$ m/s, $\quad I = 3.42 \times 10^{-3}$ m^4,

$A = 0.2025$ m^2, $\quad \alpha = 1.18$

This is obviously a thick beam.

Fig. 7 compares the deflection of the free end of Beam 1 calculated using DET (with neglecting the rotatory inertia and shear deformation) and programs for automatic finite element calculations (PAFEC) [8]. So, in both calculations the beam is considered to be Euler–Bernoulli. It is seen that as the moving mass approaches the free end the dynamic deflection of the free end increases. The maximum difference between DET and PAFEC is 10% and it occurs when the moving mass reaches the free end of the beam. This difference will decrease by increasing the number of rigid elements. In Fig. 8 the dynamic deflection of mid-span of Beam 2 calculated by DET and finite difference method [7] are compared. The maximum dynamic deflection of the mid-point of the beam occurs when the moving mass has traveled 70% of the beam length. It is seen that the results

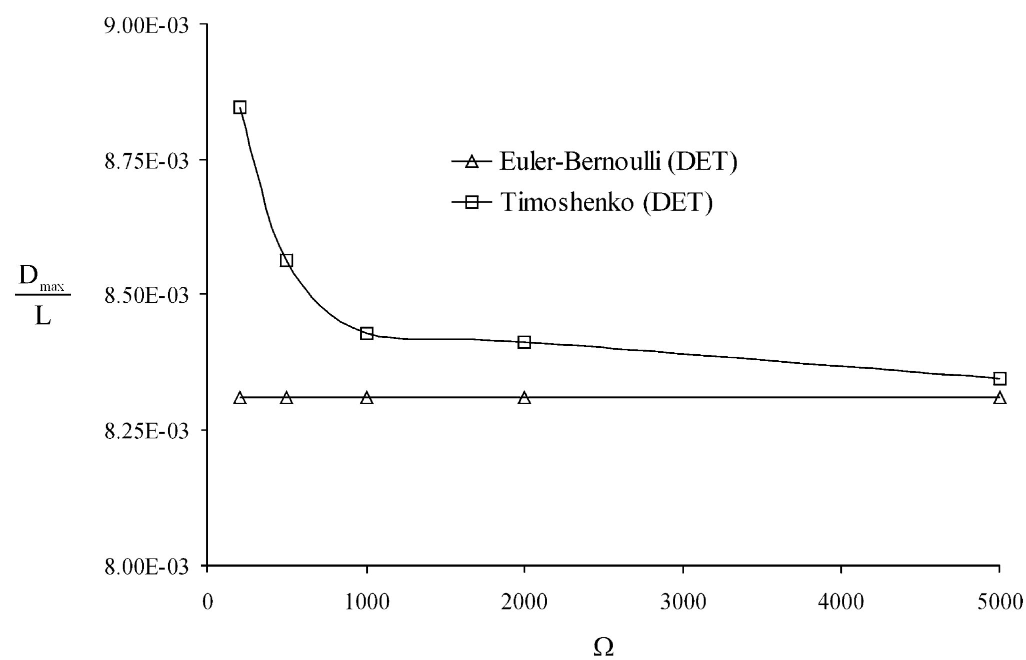

Fig. 10. Effect of beam slenderness on dynamic response of Beam 1 (D_{max} is the maximum deflection at the free end).

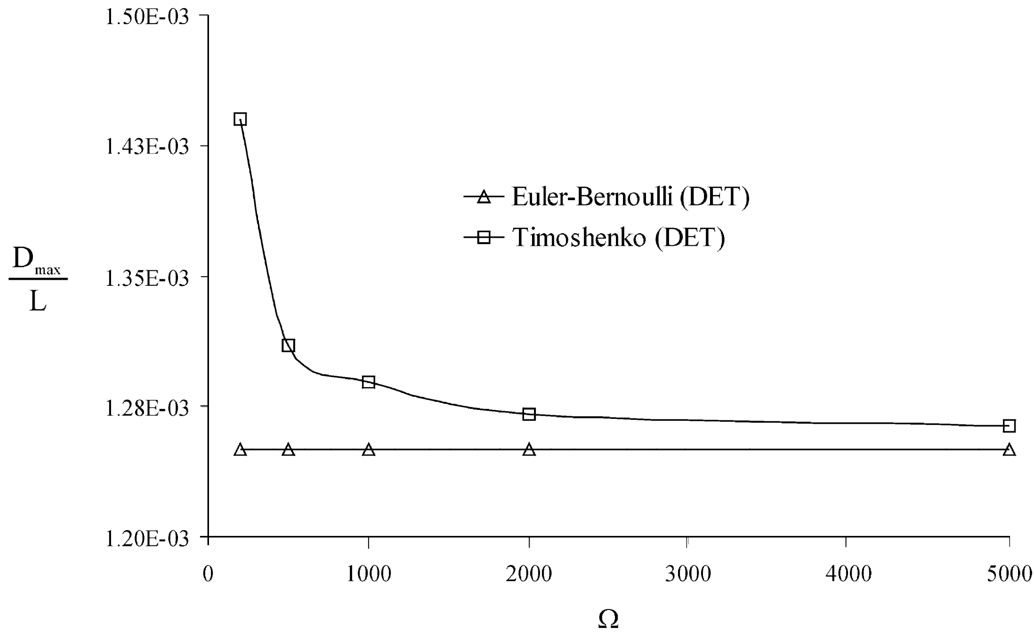

Fig. 11. Effect of beam slenderness on dynamic response of Beam 2 (D_{max} is the maximum deflection at the free end).

are fairly close and the maximum difference is 3%. Dynamic deflection of mid-span of Beam 3 according to Euler–Bernoulli and Timoshenko theories are compared in Fig. 9. As we expected, Timoshenko theory predicts larger displacements. Figs. 10 and 11 compare Euler–Bernoulli and Timoshenko theories for Beam 1 and 2. It is seen that the maximum deflection at the free end predicted by Euler–Bernoulli theory is independent of the dimensionless parameter $\Omega = AL^2/I$ (slenderness). For the Timoshenko beam this displacement increases as Ω decreases. The parameter Ω is a measure of deepness of the beam; the smaller the Ω the deeper the beam. Effect of the moving mass velocity c on deflections of Beam 1 and 2 are shown in Figs. 12 and 13. As the speed of the moving mass increases the maximum dynamic deflection of the end-point increases until it reaches a maximum. Above this speed, the maximum deflection decreases with increase in the moving mass speed. This moving mass speed that corresponds to the maximum end-point deflection is called critical speed of the moving mass. It is seen from Fig. 12 that the maximum dynamic deflection of the free end is 12% greater than the deflection of the free end when the same mass is acting at the free end statically. Fig. 13 is similar to Fig. 12.

5. Conclusions

In this paper, a DET model for Timoshenko beams under moving mass was presented. In this model, the continuous beam is replaced by a system of rigid bars and flexible joints. The flexible joints resist both relative rotation and

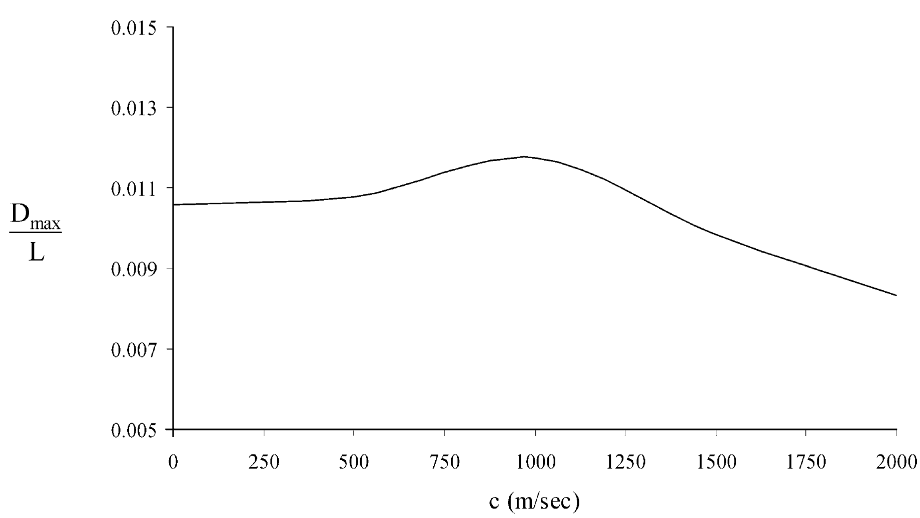

Fig. 12. Effect of increase in moving mass velocity for Beam 1 (D_{max} is the maximum deflection at the free end).

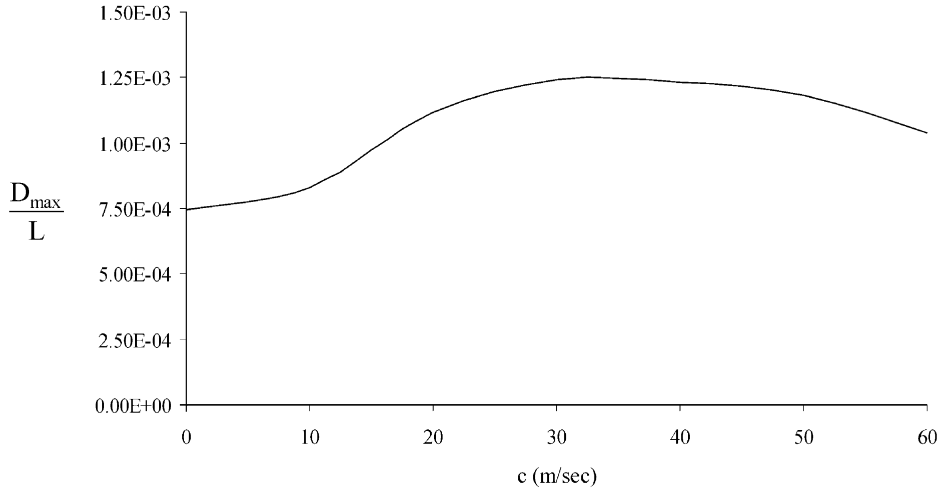

Fig. 13. Effect of increase in moving mass velocity for Beam 2 (D_{max} is the maximum deflection at the free end).

relative deflection of adjacent bars. The rotational and translational stiffnesses of the joints are functions of the material and geometrical properties of the Timoshenko beam. Some test problems were solved by the DET model and the results were compared to the results obtained by PAFEC and finite difference method. Good agreement was observed in all cases. This approximate technique can be applied to thin and thick beam structures with various boundary conditions. We formulated the problem without considering damping. However, including damping is straightforward. This method is fast and computationally inexpensive and can be used in real design problems.

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